Seminár Bratislava 24. 12. 2012

Uniform Filling of Multiparticle Phase Space

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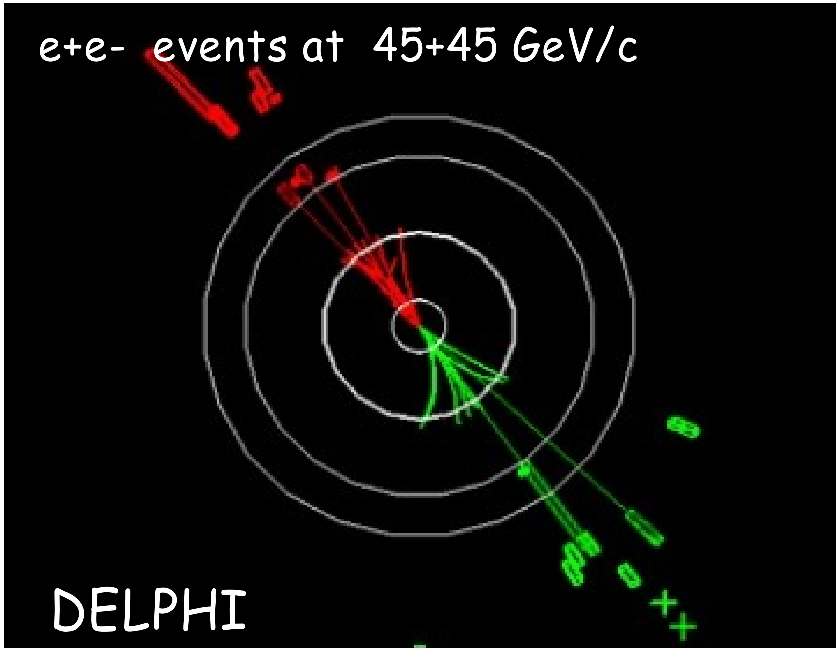
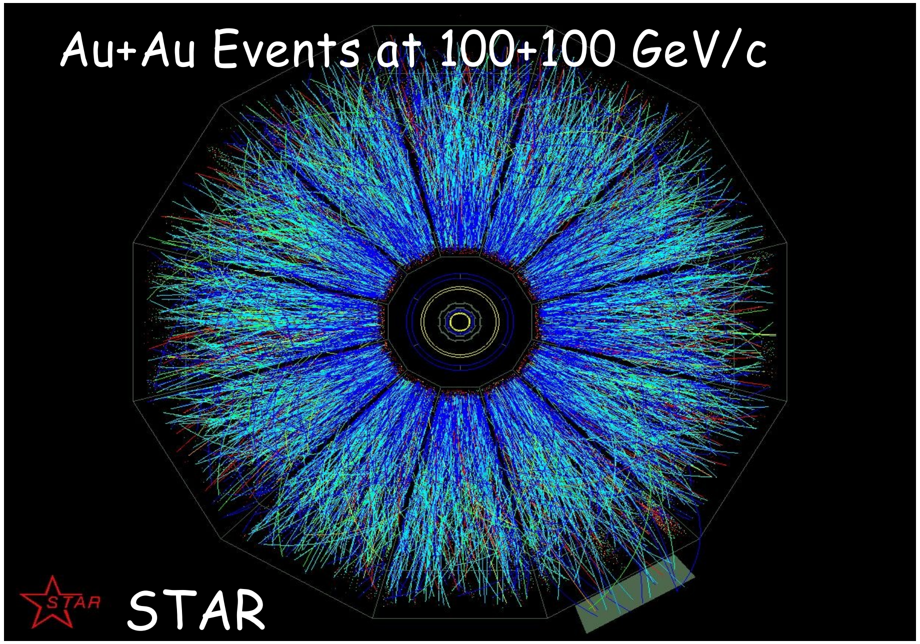
Multiparticle phase space?

e+e-, LEP



Heavy ion, RHIC



Event: E, p_x, p_y, p_z *každej častice* $E^2 = p^2 + m^2$

LEP: $n \geq 4$

LHC (pp): $n \geq 5,6,8$

LHC (Pb Pb): $n \geq 100$

Lorentz invariant phase space - LIPS

$$a + b \rightarrow 1 + 2 + 3 \cdots + n$$

$$\sigma = const \int |M|^2 d\Phi_n$$

$|M|^2$ → dynamics

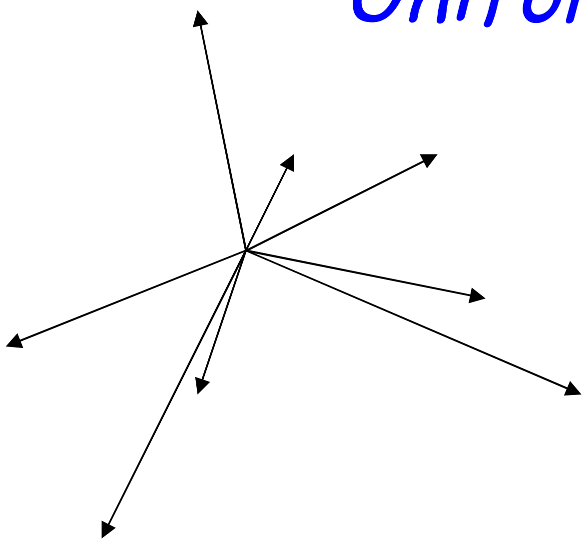
$d\Phi_n$ → kinematics & statistics

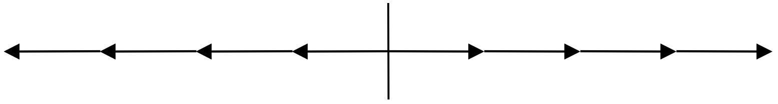
$$= const \int |M|^2 \frac{d^3\boldsymbol{p}_1}{(2\pi)^3 2E_1} \frac{d^3\boldsymbol{p}_2}{(2\pi)^3 2E_2} \cdots \frac{d^3\boldsymbol{p}_n}{(2\pi)^3 2E_n} \delta^4(p_a + p_b - p_1 - p_2 - p_3 \cdots - p_n)$$

LIPS:

$$\int d\Phi_n = \int \frac{d^3\boldsymbol{p}_1}{2\pi)^3 2E_1} \frac{d^3\boldsymbol{p}_2}{(2\pi)^3 2E_2} \cdots \frac{d^3\boldsymbol{p}_n}{(2\pi)^3 2E_n} \delta^4(p_a + p_b - p_1 - p_2 - p_3 \cdots - p_n)$$

Uniform filling of LIPS?





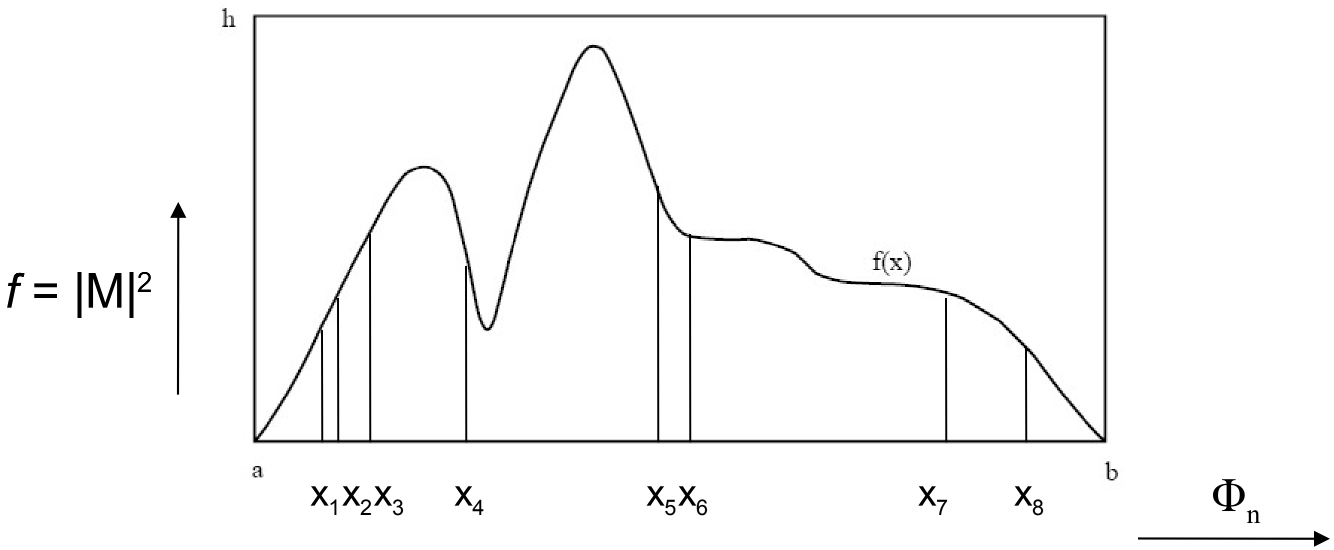
- to calculate σ by Monte Carlo integration

$$\sigma = const \int |M|^2 d\Phi_n$$

- to generate events

Monte Carlo Integration

$$\sigma = const \int |M|^2 d\Phi_n$$



Sample mean method

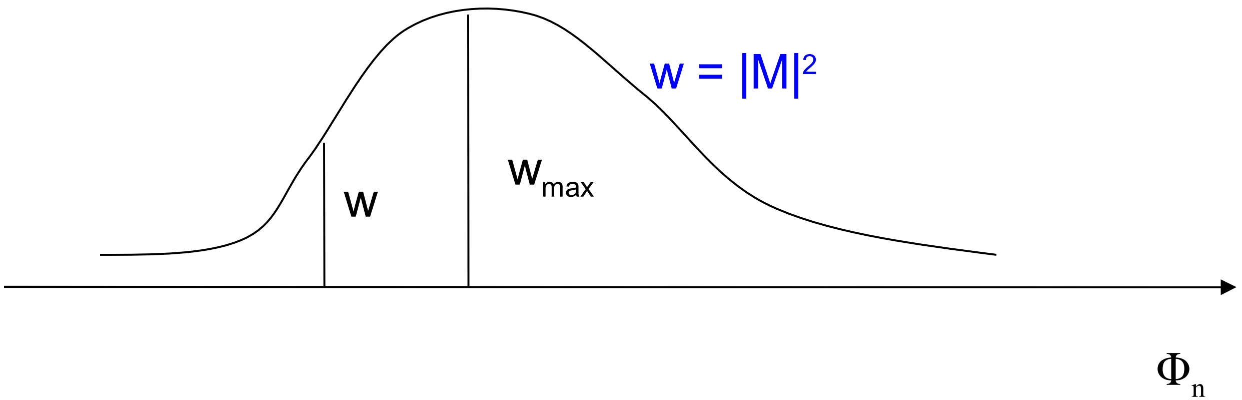
$$F_n = (b-a)\langle f \rangle = (b-a)\frac{1}{n}\sum_{i=1}^{n} f(x_i)$$

x_i have to be uniformly distributed

Event generation

$$a + b \rightarrow 1 + 2 + 3 \cdots + n$$

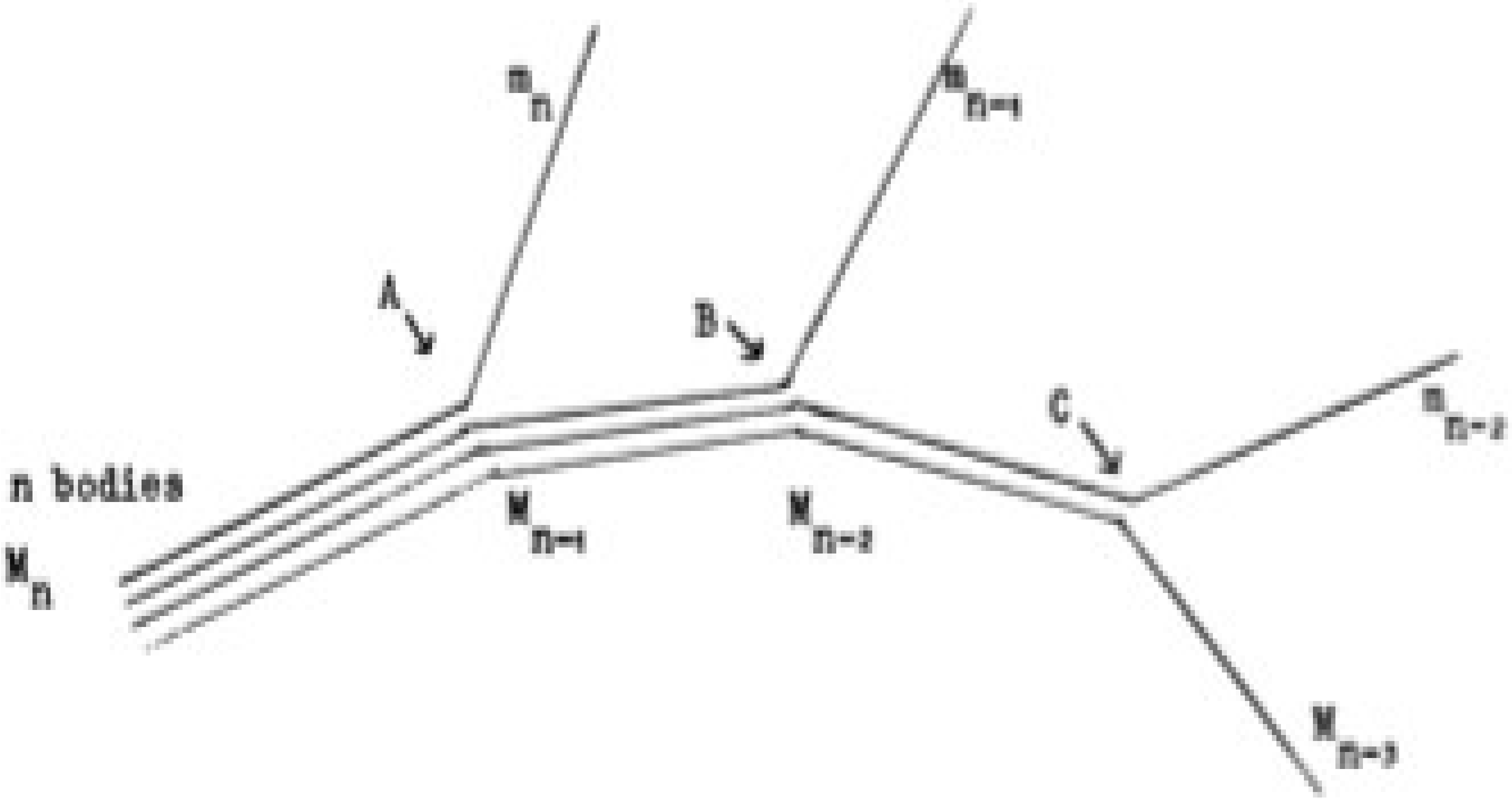
$$\sigma = const \int |M|^2 d\Phi_n$$



- Take event uniformly distributed in Φ_n
- Calculate weight $w = |M|^2$ for this event
- Accept this event with probability w/wmax

GENBOD generator (F. James)

fills LIPS uniformly



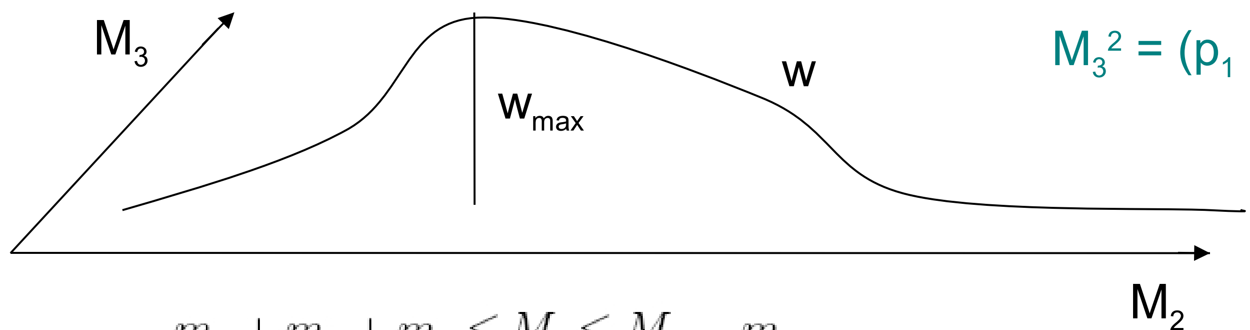
Generate events with Genbod

$$\int d\Phi_4 = \int \frac{d^3\boldsymbol{p}_4}{(2\pi)^3 2E_4} \frac{d^3\boldsymbol{p}_3}{(2\pi)^3 2E_3} \frac{d^3\boldsymbol{p}_2}{(2\pi)^3 2E_2} \frac{d^3\boldsymbol{p}_1}{(2\pi)^3 2E_1} \delta^4(p_a + p_b - p_1 - p_2 - p_3 - p_4)$$

$$= \int_{M_{3min}}^{M_{3max}} \int_{M_{2min}}^{M_{2max}} w \; dM_2 dM_3$$

$M_2^2 = (p_1 + p_2)^2$

$M_3^2 = (p_1 + p_2 + p_3)^2$



$$m_1 + m_2 + m_3 \leq M_3 \leq M_4 - m_4$$
$$m_1 + m_2 \leq M_2 \leq M_3 - m_3$$

- Generate M_2, M_3 uniformly within kinematic limits
- Calculate weight w
- Accept M_2, M_3 with probability w/wmax
- Generate angles, calculate momenta, boost to Lab system

GENBOD vs other generators

GENBOD
w/wmax very low
Good for n < 30

RAMBO
w/wmax much better, = 1 for massless particles
Good for n < 100 relativistic particles

NUPHAZ
w/wmax best so far, = 1 for massless particles
Better than RAMBO, relativistic particles

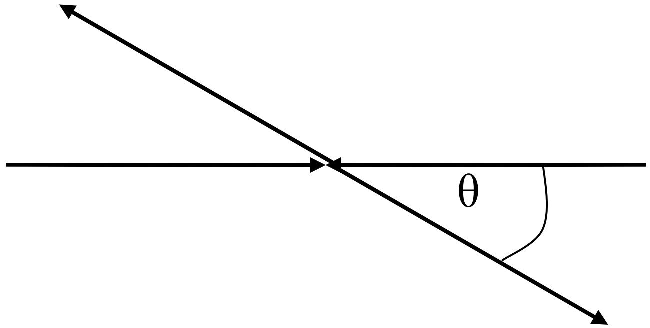
REGGAE (Tomášik, Mereš, Melo,Balek,Černý)

(REscattering after Genbod GenerAtor of Events)

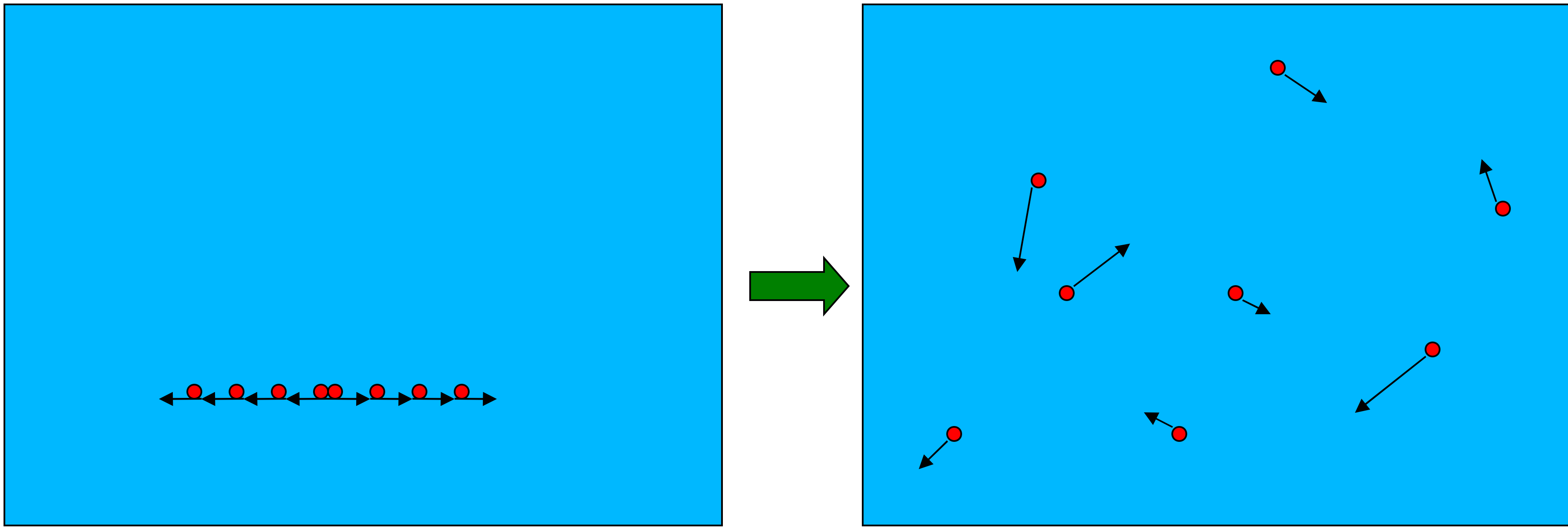
Computer Physics Communications **182** (2011) 2561-2566.

Aim to generate pure phase space events with high multiplicity and efficiency for both relativistic and nonrelativistic particles

- Use Genbod in the 1st step to generate event with any w
- Let the particles in the event collide virtually to reach the most probable configurations in the phase space with w → 1



Gas in a box



Small w

Large w

For large n → Maxwell-Boltzmann with temperature T

LIPS $\xrightarrow{\text{large n}}$ LIPS-Boltzmann

$$\int d\Phi_n = \int \frac{d^3\boldsymbol{p}_n}{(2\pi)^3 2E_n} \cdots \frac{d^3\boldsymbol{p}_3}{(2\pi)^3 2E_3} \frac{d^3\boldsymbol{p}_2}{(2\pi)^3 2E_2} \frac{d^3\boldsymbol{p}_1}{(2\pi)^3 2E_1} \delta^4(p_a + p_b - p_1 - p_2 - p_3 \cdots - p_n)$$

Darwin-Fowler method

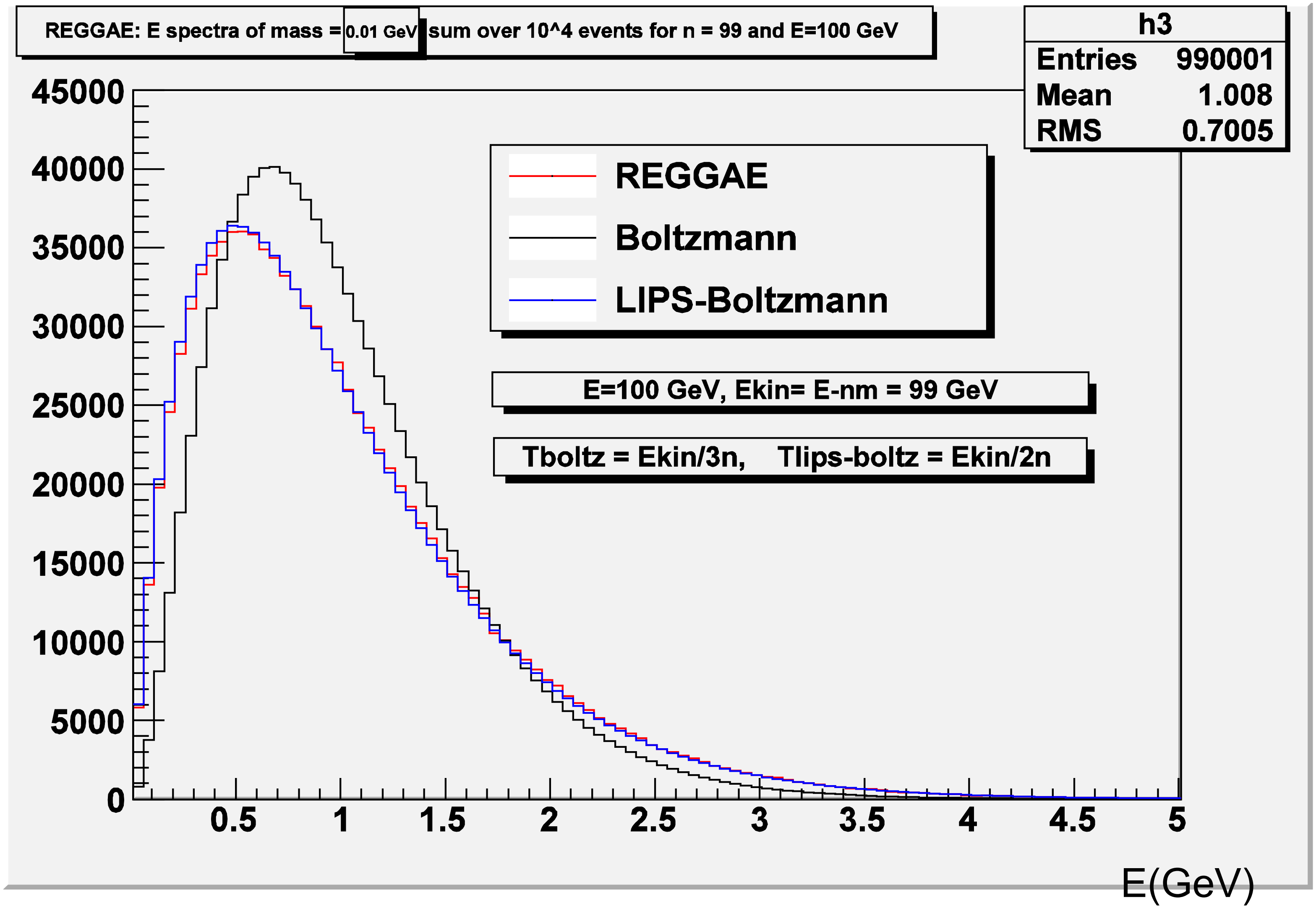
$$\rho(E)dE = N\sqrt{E^2 - m^2}\exp\left(-\frac{E}{kT}\right)dE$$

Microcanonical ensemble $\xrightarrow{\text{large n}}$ Boltzmann

$$\int d\Phi_n = \int \frac{d^3\boldsymbol{p}_n}{(2\pi)^3 \square} \cdots \frac{d^3\boldsymbol{p}_3}{(2\pi)^3 \square} \frac{d^3\boldsymbol{p}_2}{(2\pi)^3 \square} \frac{d^3\boldsymbol{p}_1}{(2\pi)^3 \square} \delta^4(p_a + p_b - p_1 - p_2 - p_3 \cdots - p_n)$$

$$\rho(E)dE = N\sqrt{E^2 - m^2}\exp\left(-\frac{E}{kT}\right)dE$$

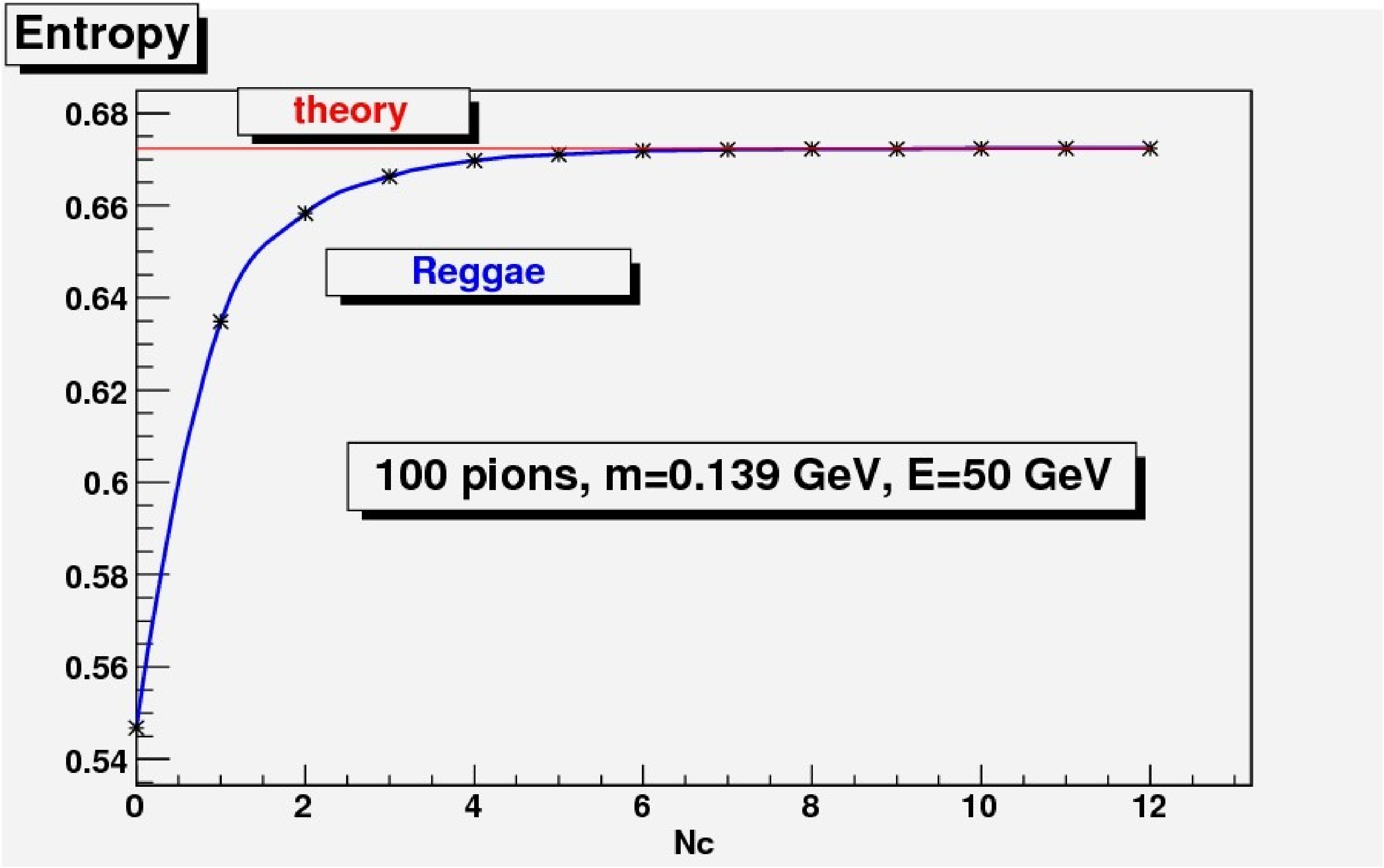
E



REGGAE: E spectra of mass = 0.01 GeV sum over 10^4 events for n = 99 and E=100 GeV
h3
Entries 990001
Mean 1.008
RMS 0.7005
REGGAE
Boltzmann
LIPS-Boltzmann
E=100 GeV, Ekin= E-nm = 99 GeV
Tboltz = Ekin/3n, Tlips-boltz = Ekin/2n
45000
40000
35000
30000
25000
20000
15000
10000
5000
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
E(GeV)

Information entropy

$$S = -\int_{\Sigma} \rho(\boldsymbol{p}) \, \ln \rho(\boldsymbol{p}) \, d^3\boldsymbol{p}$$



REGGAE vs other generators

numerical integration

$$\sigma = const \int |M|^2 d\Phi_n$$

$$I = \int_{\Phi_n} f(\{p_i\})\, d\Phi_n \,, \qquad \{p_i\} = (p_1, p_2, p_3, \ldots, p_n)$$

$$\hat{I} = \left(\frac{1}{N} \sum_{j=1}^{N} f(\{p_i\}_j) \right) \Phi_n$$

$$f_5(p_1, p_2, p_3, p_4, p_5) = \frac{(p_1^2 + p_2^2 + p_3^2) p_1^2}{M^4 + p_4^2 p_5^2}$$

n = 30 particles with mass 1 GeV, p_a+p_b = (100 GeV, 0, 0, 0)

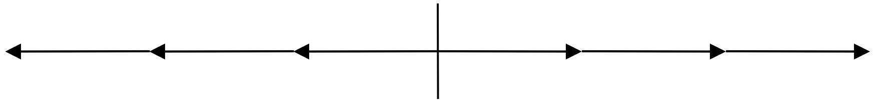
N	REGGAE $N_c = 3$	REGGAE $N_c = 4$	REGGAE $N_c = 6$	NUPHAZ	RAMBO	wGENBOD
10^4	13.63	13.41	13.78	12.77	13.23	12.23
10^5	13.99	13.52	13.36	13.15	13.21	-
10^6	13.84	13.42	13.19	13.06	13.12	-
time	16 min	20 min	28 min	6 min	11 min	300 min

n = 60 particles with mass 1 GeV

N	REGGAE ($N_c = 6$)	NUPHAZ
10^4	0.6339	0.6185
10^5	0.6185	0.6315
time for 10^5	6 min	63 min

What next?

REGGAE can fill LIPS uniformly for fixed n and chemical composition



… but can we predict if total CM energy prefers to convert to, say, n=50 or n=60 particles? I.e. generate events with different n?

Question: Molecules colliding in a box give canonical Boltzmann, how is this different from REGGAE collisions which give LIPS-Boltzmann?

Answer: Molecules in a box collide in both momentum and configuration space

Question: Can we adjust REGGAE collisions to get canonical Boltzmann?

Question: If we get canonical Boltzmann, do we also get uniform filling of the microcanonical phase space?

What is the difference between LIPS and microcanonical phase space?

- LIPS counts states in the momentum space, these states are asymptotic (infinite volume)

$$\int d\Phi_n = \int \frac{d^3\boldsymbol{p}_n}{(2\pi)^3 2E_n} \cdots \frac{d^3\boldsymbol{p}_3}{(2\pi)^3 2E_3} \frac{d^3\boldsymbol{p}_2}{(2\pi)^3 2E_2} \frac{d^3\boldsymbol{p}_1}{(2\pi)^3 2E_1} \delta^4(p_a + p_b - p_1 - p_2 - p_3 \cdots - p_n)$$

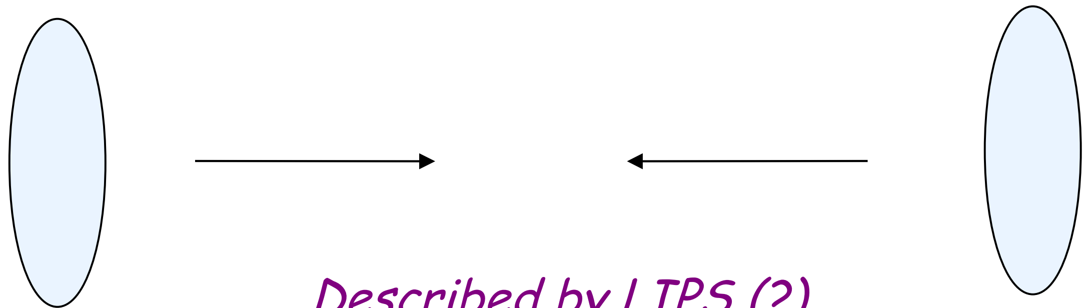
- Microcanonical counts states both in momentum and configuration space

which has finite volume V

$$\int d\Phi_n = \int \frac{d^3\boldsymbol{p}_n}{(2\pi)^3 \boxed{2E_n}} \cdots \frac{d^3\boldsymbol{p}_3}{(2\pi)^3 \boxed{2E_3}} \frac{d^3\boldsymbol{p}_2}{(2\pi)^3 \boxed{2E_2}} \frac{d^3\boldsymbol{p}_1}{(2\pi)^3 \boxed{2E_1}} \delta^4(p_a + p_b - p_1 - p_2 - p_3 \cdots - p_n) \; V^n$$

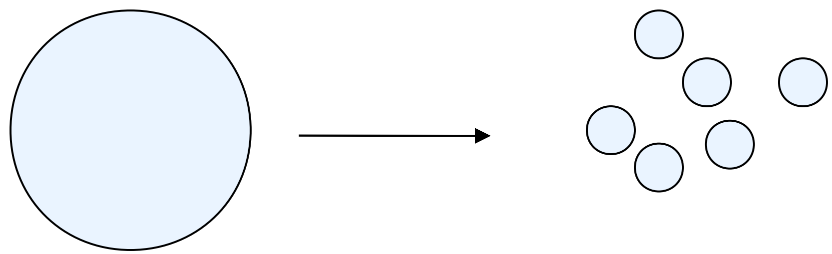
Statistical model of hadronization

Heavy ion collision



Described by LIPS (?)

fireball



hadrons

finite V

Described by microcanonical phase space

BACKUP

GENBOD generator (F. James)

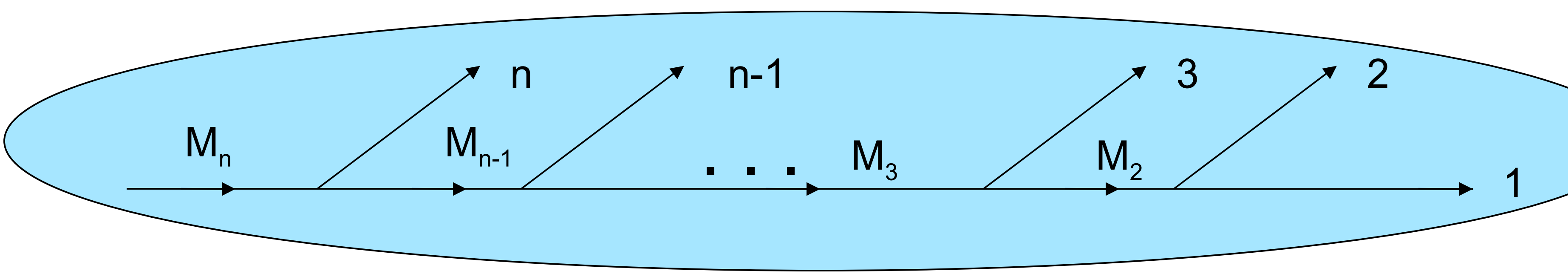
$M_n \rightarrow 1 + 2 + 3 + \ldots + n$ $\quad M_n^2 = (p_a + p_b)^2$

$$\int d\Phi_n = \int \frac{d^3\boldsymbol{p}_n}{(2\pi)^3 2E_n} \cdots \frac{d^3\boldsymbol{p}_3}{(2\pi)^3 2E_3} \frac{d^3\boldsymbol{p}_2}{(2\pi)^3 2E_2} \frac{d^3\boldsymbol{p}_1}{(2\pi)^3 2E_1} \delta^4(p_a + p_b - p_1 - p_2 - p_3 \cdots - p_n)$$

$M_2 \rightarrow 1 + 2$ $\quad M_2^2 = (p_1 + p_2)^2$

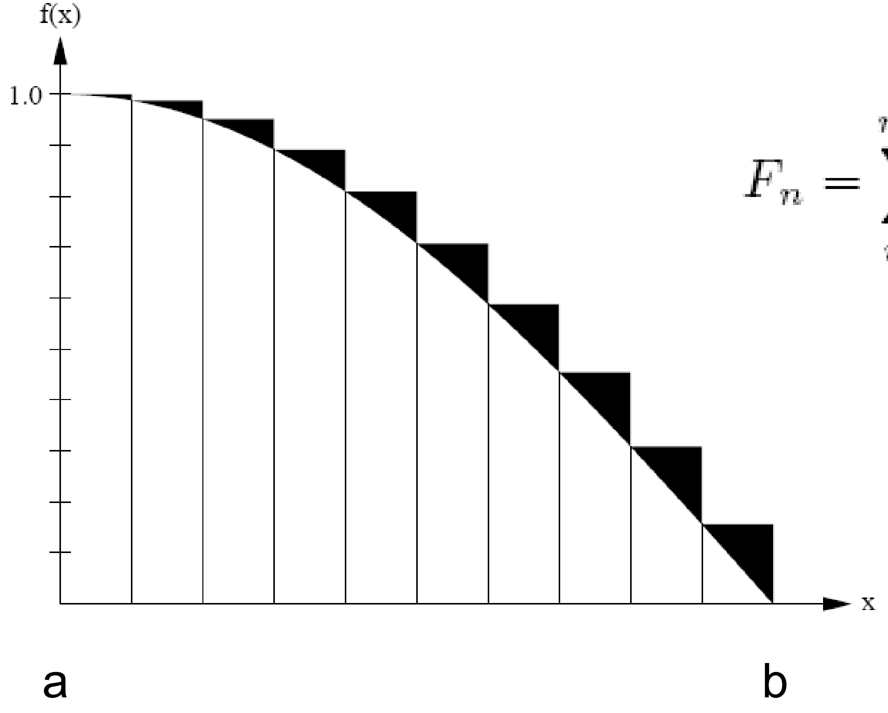
$M_3 \rightarrow 1 + 2 + 3 \rightarrow 3 + M_2$ $\quad M_3^2 = (p_1 + p_2 + p_3)^2$

Each 2-body decay evaluated in the CM frame of 2 daughters



Standard Numerical Methods of Integration

$$F = \int_a^b f(x)\,dx.$$



$$F_n = \sum_{i=0}^{n-1} f(x_i)\Delta x. \qquad \Delta x = \frac{b-a}{n}$$

Rectangular

Trapezoidal

Simpson

GENBOD generator (F. James)

$$\int d\Phi_4 = \int \frac{d^3\boldsymbol{p}_4}{(2\pi)^3 2E_4} \frac{d^3\boldsymbol{p}_3}{(2\pi)^3 2E_3} \frac{d^3\boldsymbol{p}_2}{(2\pi)^3 2E_2} \frac{d^3\boldsymbol{p}_1}{(2\pi)^3 2E_1} \delta^4(p_a + p_b - p_1 - p_2 - p_3 - p_4)$$

$\Downarrow$

$$\int d\Phi_4 = 2^3 \int_{M_{3min}}^{M_{3max}} M_3 R_2(M_4, M_3, m_4) \int_{M_{2min}}^{M_{2max}} M_2 R_2(M_3, M_2, m_3) R_2(M_2, m_1, m_2) dM_2 dM_3$$

where $M_2^2 = (p_1 + p_2)^2$ $M_3^2 = (p_1 + p_2 + p_3)^2$

$$R_2(M_4, M_3, m_4) = \frac{2\pi}{M_4}\sqrt{M_4^2 + \left(\frac{M_3^2 - m_4^2}{M_4}\right)^2 - 2(M_3^2 + m_4^2)}$$

$$m_1 + m_2 + m_3 \leq M_3 \leq M_4 - m_4$$
$$m_1 + m_2 \leq M_2 \leq M_3 - m_3$$

12 variables $\rightarrow$ 2

Pure phase space
(Lorentz Invariant Phase Space, LIPS)

$$\sigma = const \int |M|^2 d\Phi_n$$ ... complicated

→ numerically

→ $|M|^2 \longrightarrow 1$

$$\int d\Phi_n = \int \frac{d^3\boldsymbol{p}_1}{(2\pi)^3 2E_1} \frac{d^3\boldsymbol{p}_2}{(2\pi)^3 2E_2} \cdots \frac{d^3\boldsymbol{p}_n}{(2\pi)^3 2E_n} \delta^4(p_a + p_b - p_1 - p_2 - p_3 \cdots - p_n)$$

Pure multiparticle phase space = LIPS

kinematics & statistics

Standard methods vs MC

(Error scaling with n)

Number of dimensions	Standard methods			Monte Carlo
	Rectangular	Trapezoidal	Simpson	
1	$1/n$	$1/n^2$	$1/n^4$	$1/\sqrt{n}$
2	$1/\sqrt{n}$	$1/n$	$1/n^2$	$1/\sqrt{n}$
d	$\frac{1}{n^{1/d}}$	$\frac{1}{n^{2/d}}$	$\frac{1}{n^{4/d}}$	$1/\sqrt{n}$

Eur. J. Phys. 4 (1983) 68–71. Printed in Northern Ireland

A simple computer simulation of molecular collisions leading to Maxwell distribution

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Abstract We describe a simple computer program which simulates molecular collisions in two dimensions and leads to Maxwell distribution. The results show that even with 5–10 colliding molecules the velocity distribution is quite close to Maxwell's.

Zusammenfassung Ein einfaches Computerprogramm wird beschrieben, das molekulare Stöße in zwei Dimensionen simuliert und zur Maxwell-Verteilung führt. Die Ergebnisse demonstrieren, daß bereits mit Stößen von 5–10 Molekülen nahezu Maxwell-Geschwindigkeitsverteilung ereicht wird.