Prednáška č. 12 9.5.2022

Statická rovnováha tuhého telesa

1. Výslednica vonkajších síl musí byť nulová

$$\sum_{i=1}^{n} \vec{F_i} = 0$$

$v_0 = 0$

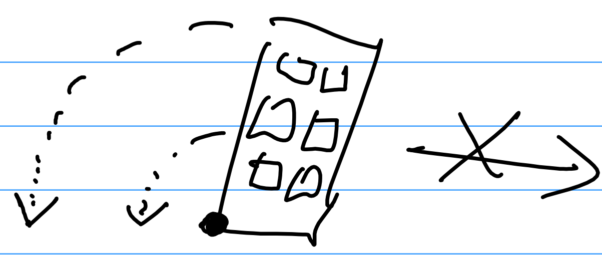
translačná rovnováha

2. Výslednica vonkajších momentov síl musí byť nulová (okolo ľubovoľného počiatku)

$$\sum_{i=1}^{n} \vec{M_i} = 0$$

rotačná rovnováha

$\omega_0 = 0$

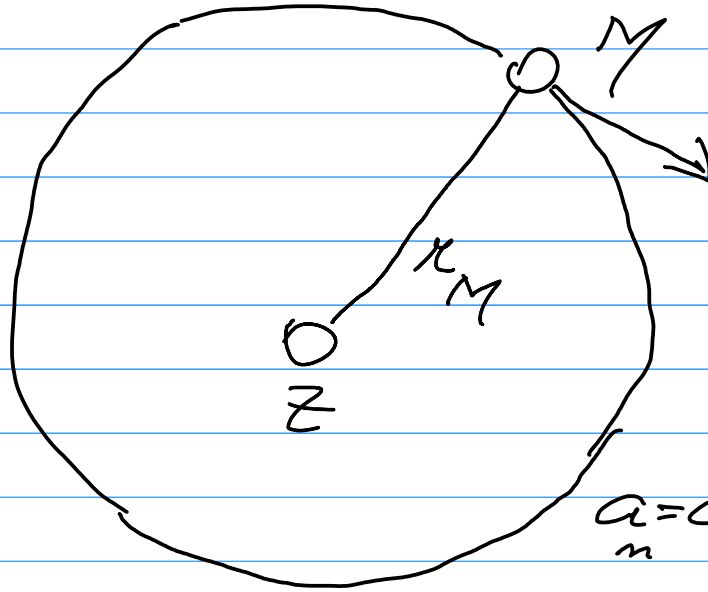


Newtonov gravitačný zákon

r. 1687 Isaac Newton

$$F_g = k\frac{1}{r^2}$$

$$= k'\frac{m}{r^2}$$



$$v = \frac{2\pi r_M}{T}$$

$$a = a_n = a_M = \frac{v^2}{r_M}$$

$$a_M = \frac{4\pi^2 \cancel{r_M^2}}{T^2 \cancel{r_M}}$$

$$a_M = \frac{4\pi^2}{T^2} r_M$$

$T = 27{,}32$ dňa

$T = 2{,}36 \cdot 10^6 s$

$r_M = 3{,}84 \cdot 10^8 m$

$$a_M = 2{,}72 \cdot 10^{-3} ms^{-2}$$

$$F_g^M = k'\frac{M_M}{r_M^2}$$

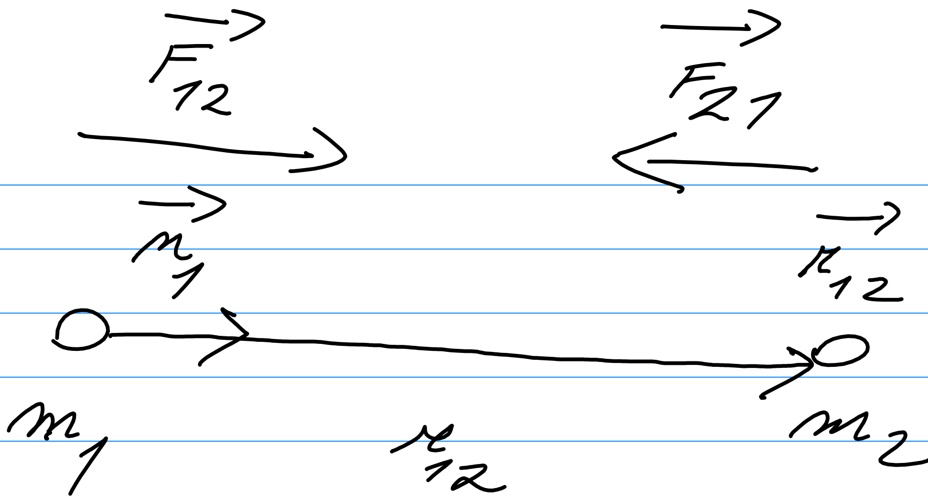
$$F_g^{jablko} = k'\frac{M_{jablko}}{r_Z^2}$$

$$a_M = \frac{F_g^M}{M_M} = \boxed{\frac{k'}{r_M^2}}$$

$$a_{jablko} = \frac{F_g^{jablko}}{M_{jablko}} = \frac{k'}{r_Z^2} = g$$

$$\frac{a_M}{g} = \frac{\frac{k'}{r_M^2}}{\frac{k'}{r_Z^2}} = \frac{r_Z^2}{r_M^2} \Rightarrow a_M = \frac{r_Z^2}{r_M^2} g$$

$$= 2{,}70 \cdot 10^{-3} ms^{-2}$$



$$\vec{F}_{12} = -\vec{F}_{21}$$

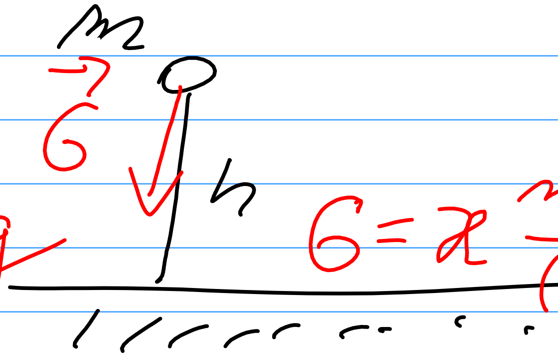
Newtonov gravitačný zákon

$$\vec{F}_{12} = \varkappa \frac{m_1 m_2}{r_{12}^2} \vec{r}_1$$

$\varkappa = 6{,}672 \cdot 10^{-11}\ Nm^2kg^{-2}$

gravitačná konštanta

Tiažová sila a gravitačná sila



$h \ll R_Z$

$G = mg$

$$G = \varkappa \frac{m M_Z}{(R_Z + h)^2}$$

$$\cancel{m}g = \varkappa \frac{\cancel{m} M_Z}{(R_Z+h)^2}$$

tiažové pole Zeme

$$g = \varkappa \frac{M_Z}{(R_Z+h)^2} = \text{konšt} = 9{,}806\ ms^{-2}$$

Na úrovni mora: $h \to 0$

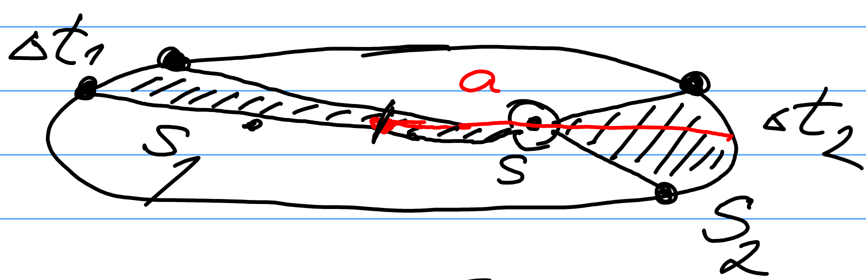
$$g = \varkappa \frac{M_Z}{R_Z^{\,2}} = 9{,}806\ ms^{-2}$$

Odváženie Zeme

$$M_Z = g \frac{R_Z^{\,2}}{\varkappa} = 5{,}98 \cdot 10^{24}\ kg$$

Keplerove zákony

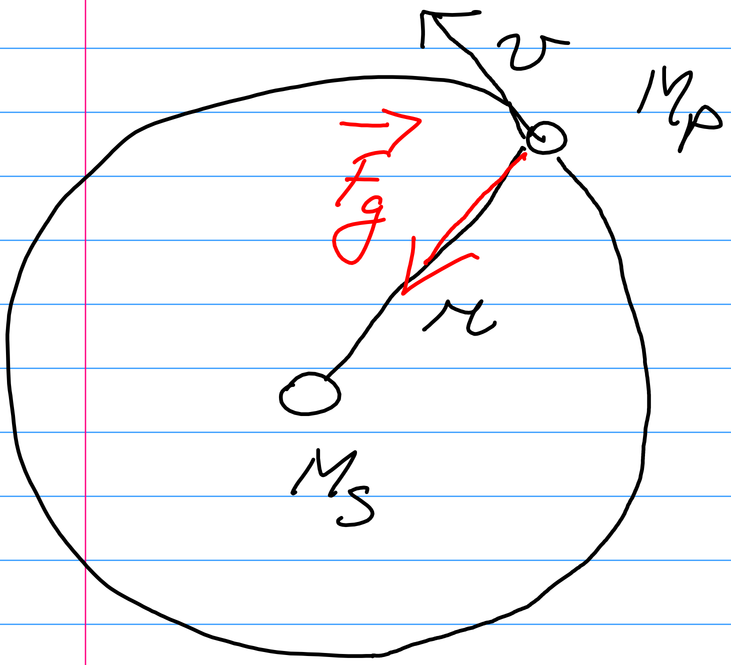
1. Planéty obiehajú okolo Slnka po elipsách (Slnko je v 1 ohnisku elipsy)

2. 

ak $\Delta t_1 = \Delta t_2$
$\Rightarrow S_1 = S_2$

3. $T^2 = k a^3$

Odvodenie 3. KZ z Newtonovho zákona



$$F_n = \frac{M_p v^2}{r}$$

$$F_g = \varkappa \frac{M_p M_S}{r^2}$$

$$v = \frac{2\pi r}{T}$$

$$F_n = F_g$$

$$\frac{\cancel{M_p} 4\pi^2 r^2}{r T^2} = \varkappa \frac{\cancel{M_p} M_S}{r^2}$$

$$K \quad \frac{4\pi^2 r^3}{\varkappa M_S} = T^2$$

$$\boxed{T^2 = K r^3} \equiv \underline{3.\,KZ}$$

Gravitačná potenciálna energia

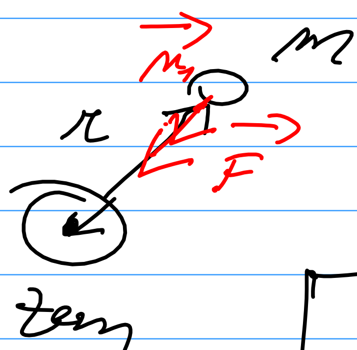
$$\Delta W_p = W_{p_2} - W_{p_1} = -\int_1^2 \vec{F} \cdot d\vec{r} = -A$$

tiažové pole Zeme

$$W_p = mgh$$

gravitačné pole:

$$\vec{F} = \varkappa \frac{m M_Z}{r^2} \vec{n}_1$$



$r = R_Z + h$

$$\boxed{W_p = -\varkappa \frac{m M_Z}{r}}$$

Zákon zachovania celkovej mechanickej energie

$$W_{k_1} + W_{p_1} = W_{k_2} + W_{p_2}$$

$$\frac{1}{2} m v_1^2 - \varkappa \frac{m M_Z}{r_1} = \frac{1}{2} m v_2^2 - \varkappa \frac{m M_Z}{r_2}$$

úniková rýchlosť z gravitačného poľa

$$r_1 = R_Z \quad v_2 = 0, \quad r_2 = \infty$$

$$\frac{1}{2} \cancel{m} v_1^2 - \varkappa \frac{\cancel{m} M_Z}{r_1} = 0$$

$$\frac{1}{2} v_1^2 - \varkappa \frac{M_Z}{R_Z} = 0 \qquad / \cdot 2$$

$$v_1^2 = 2\varkappa \frac{M_Z}{R_Z} \qquad / \sqrt{}$$

$$v_1 = \sqrt{2\varkappa \frac{M_Z}{R_Z}} = 11{,}2\ \mathrm{km\,s^{-1}}\ \text{(Zem)}$$

$$v_1 = \quad = 2{,}3\ \mathrm{km\,s^{-1}}\ \text{(Mesiac)}$$

$$v_1 = \quad = 60\ \mathrm{km\,s^{-1}}\ \text{(Jupiter)}$$