Gravitačné pole

Od Kopernika až po Keplerove zákony
Keplerove zákony
Newtonov gravitačný zákon
Veličiny charakterizujúce gravitačné pole
- *intenzita, potenciálna energia, potenciál*

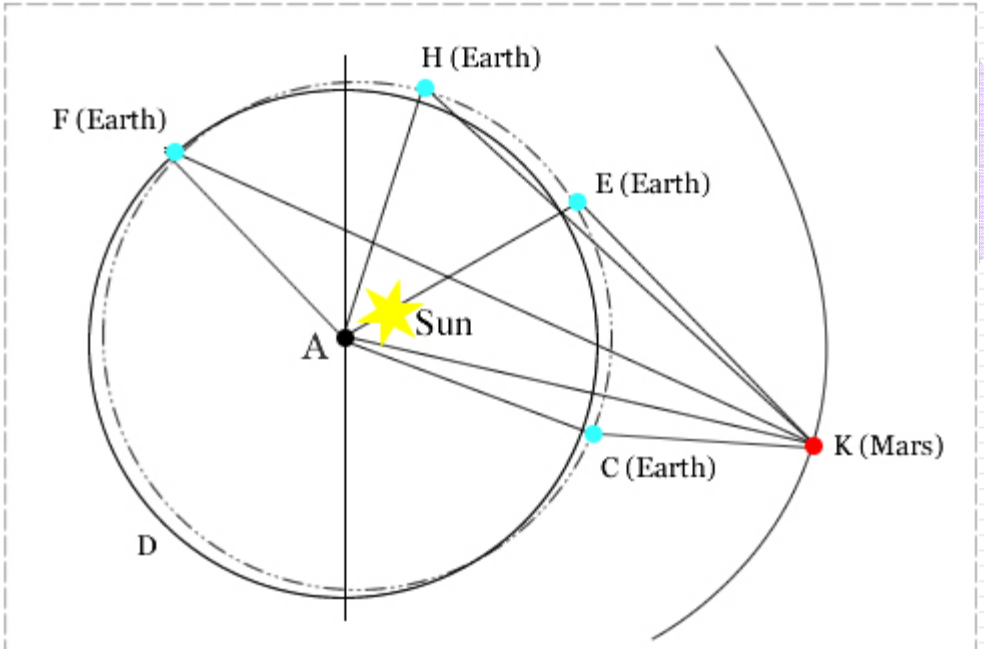
Pohyb telies v gravitačnom poli Slnka
- *1. a 2. kozmická rýchlosť*

Od geocentrizmu k heliocentrizmu

geocentrický názor

Ptolemaios + Aristoteles

Tycho Brahe (1546-1601)
Planéty sa pohybujú okolo Slnka, ale spolu so Slnkom rotujú okolo Zeme, ktorá je stredom vesmíru



heliocentrický názor

Nicolaus Copernicus (1473-1543)
Zem sa točí okolo osi aj okolo Slnka, planéty sa pohybujú okolo Slnka rovnakým smerom.

Giordano Bruno (1548-1600)
Kopernikus+ navyše ani Slnko nie je stred vesmíru ale len jedna z hviezd

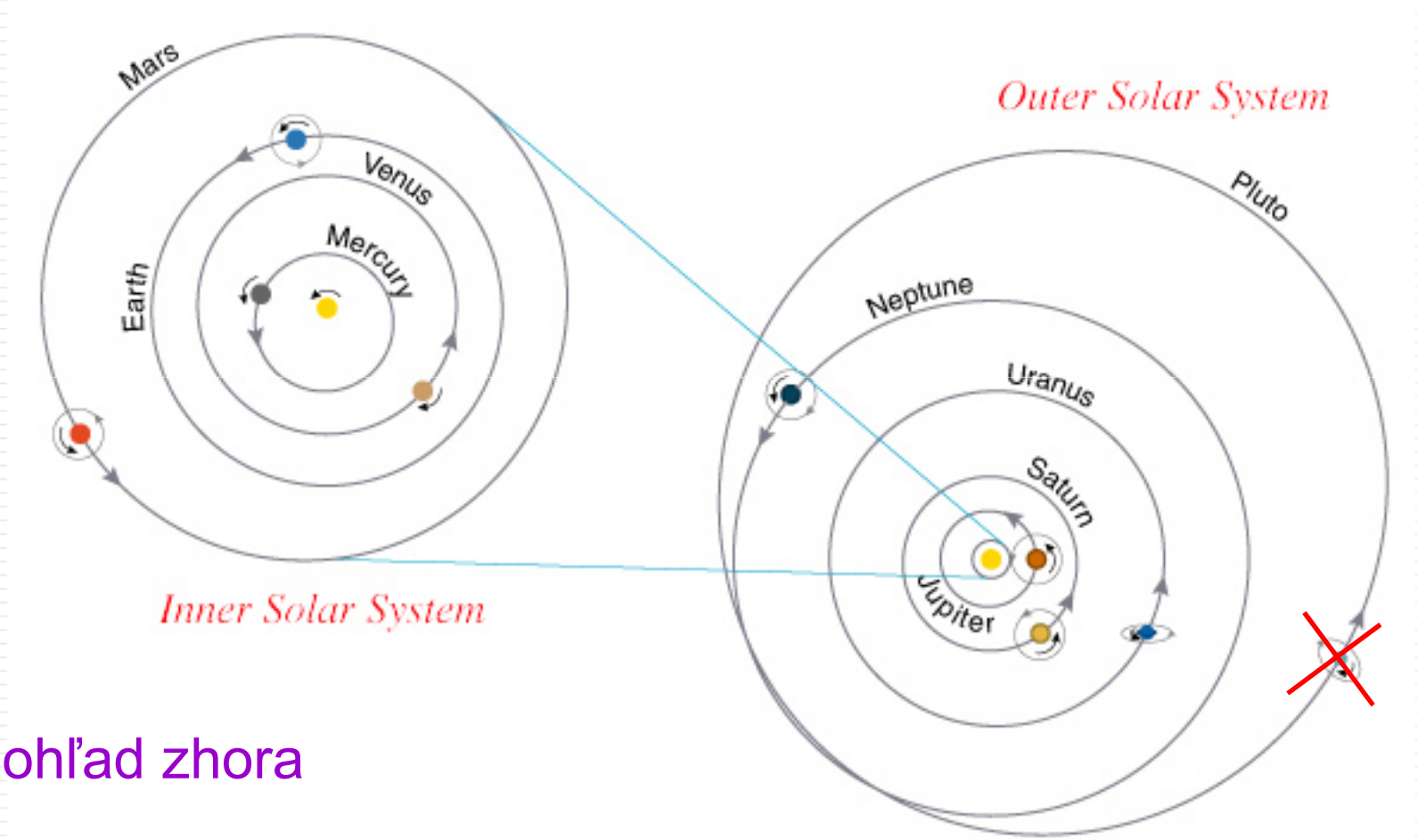
Johannes Kepler (1571-1630)
zhrnul pozorovania TB a NC a **sformuloval Keplerove zákony**, ako prvý dobre vysvetlil nezrovnalosti v pozorovaniach tým, že zaviedol eliptické dráhy

Galileo Galilei (1564-1642)
zostrojil ďalekohľady s 30x zväčš., podporil ešte viac heliocentrizmus

http://www.keplersdiscovery.com

1. Keplerov zákon I

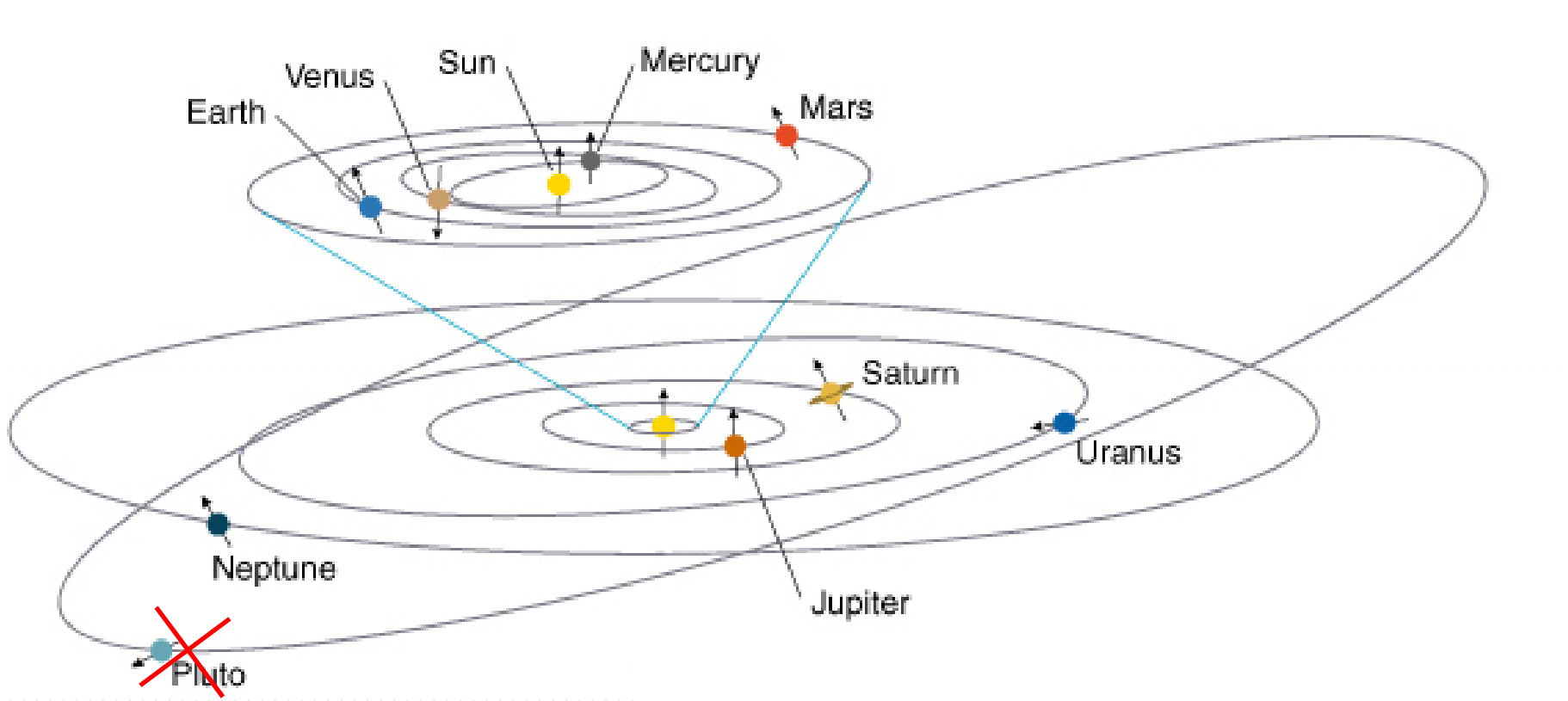
Všetky planéty obiehajú okolo Slnka po eliptických dráhach, pričom Slnko je v ich spoločnom ohnisku



pohľad zhora

Pozn. elipsa má len málo odlišné dlžky polosí (napr. Zem je v Apheliu 4. júla – 152.000.000km a v Periheliu 3. januára – 149.600.000km)

Pluto bolo v roku 2006 vyradené zo zoznamu planét Slnečnej sústavy!



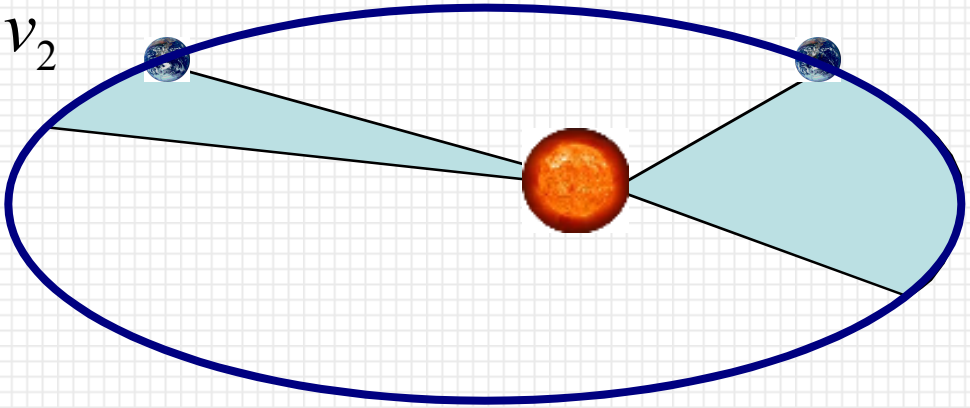
pohľad v rovine obežných dráh

Parametre planét a ich obežných dráh

	Perihelion [AU]	Aphelion [AU]	Period [day]	Mass [10^{24}kg]
Mercury	0.31	0.47	88,96	0,335
Venus	0.72	0.73	224,54	4,89
Earth	0.98	1.02	365,25	5,98
Mars	1.38	1.67	687,50	0,646
Jupiter	4.95	5.45	4328,7	1900
Saturn	9.02	10.0	10822	569
Uranus	18.3	20.1	30555	87,3
Neptune	30.0	30.3	60417	103
Pluto	29.7	49.9	90509	5,4

1AU… priemerná vzdialenosť Zem-Slnko 152.000.000km

2. Keplerov zákon



$$v_1 > v_2$$

Sprievodič spájajúci Slnko s planétou opisuje za rovnaký čas rovnaké plochy

3. Keplerov zákon

$$\frac{T_1^2}{r_1^3} = \frac{T_2^2}{r_2^3}$$

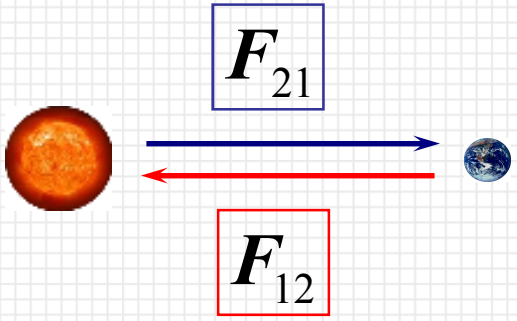
$$\frac{T^2}{r^3} = konšt.$$

(pomocou 3. Keplerovho zákona)

$$a_n = \frac{v^2}{r} = \omega^2 r = \left(\frac{2\pi}{T}\right)^2 r = r\frac{4\pi^2}{T^2} = 5{,}929.10^{-3}\, ms^{-2}$$

3.K.z.

$$T^2 = 4\pi^2 \frac{r}{a} \Rightarrow 4\pi^2 \frac{r}{ar^3} = konšt.$$



$$a = k\frac{1}{r^2}$$

$$F_{12} = ma = k\frac{m}{r^2} \qquad F_{21} = Ma' = k'\frac{M}{r^2}$$

$$k = M\kappa \qquad k' = m\kappa$$

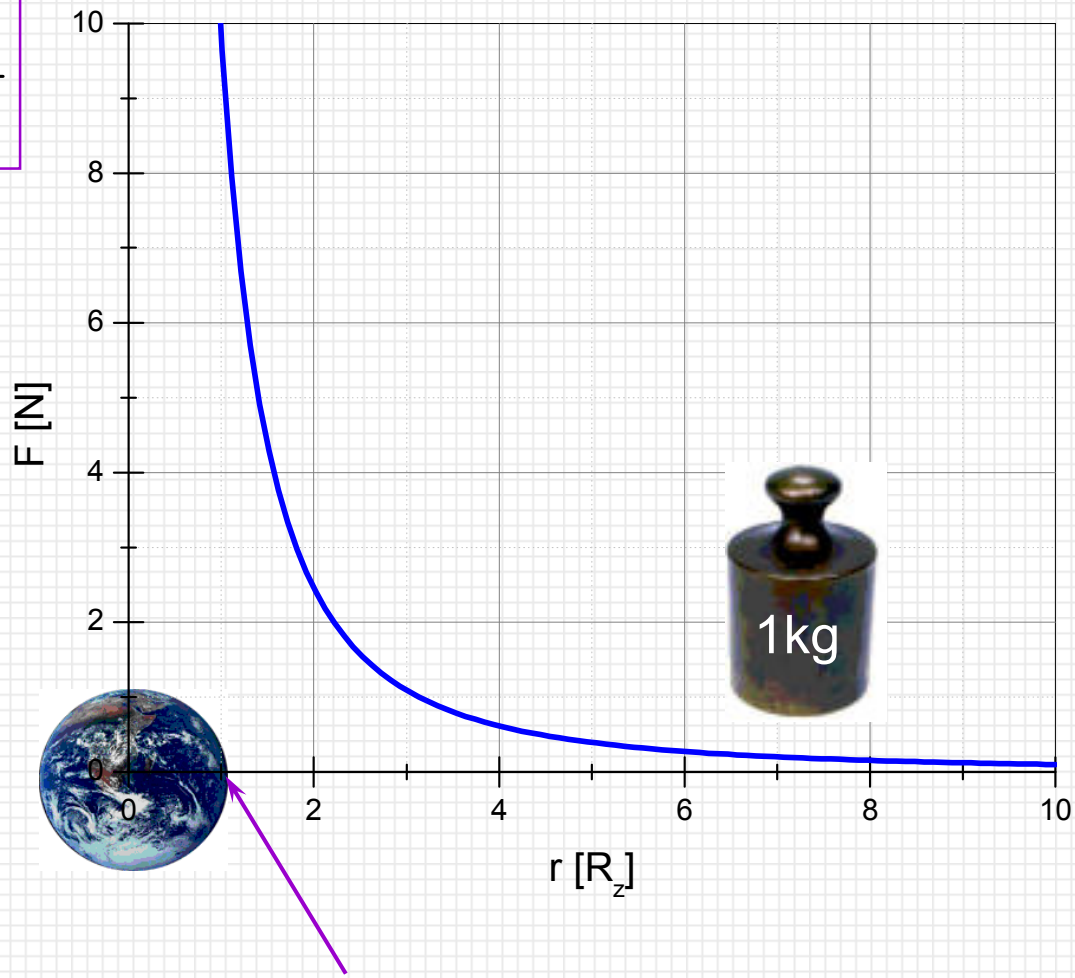
$$\boldsymbol{F}_{12} = -\kappa\frac{Mm}{r^3}\boldsymbol{r}_{12}$$

$$\kappa = 6{,}671.10^{-11}\, Nm^2kg^{-2}$$

Gravitačná sila na Zemi

Závislosť veľkosti gravitačnej sily od vzdialenosti od Zeme na hm. b. 1kg

$$F = \kappa \frac{Mm}{r^2}$$



Gravitačná sila na Zemi a na Mesiaci

Porovnanie veľkosti gravitačnej sily na povrchu Mesiaca a Zeme

$m = 1kg$

$$F = \kappa \frac{Mm}{R^2}$$

$$M_M = 7{,}349.10^{22}\,kg$$
$$R_M = 1{,}738.10^{6}\,m$$

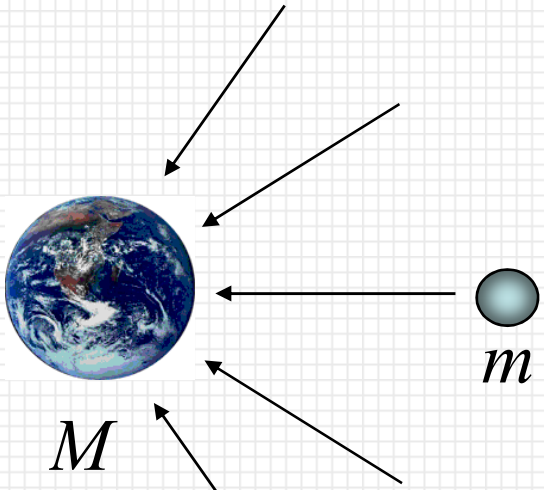
$$M_Z = 5{,}974.10^{24}\,kg$$
$$R_Z = 6{,}371.10^{6}\,m$$

$$F_M = 1{,}62N$$

$$F_Z = 9{,}81N$$

$$\frac{F_Z}{F_M} \cong 6$$

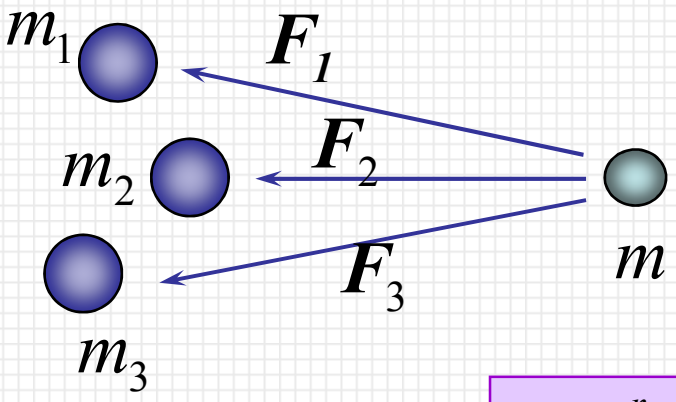
Intenzita gravitačného poľa



$$\boldsymbol{K} = \frac{\boldsymbol{F}}{m} \quad \Rightarrow \quad \boldsymbol{K} = -\kappa \frac{M}{r^3}\boldsymbol{r}$$

$$\boldsymbol{F} = -\kappa \frac{Mm}{r^3}\boldsymbol{r}$$

Intenzita v okolí sústavy hm. b. a telesa



$$\boldsymbol{K} = \frac{\boldsymbol{F}}{m} = \frac{\boldsymbol{F}_1 + \boldsymbol{F}_2 + \ldots + \boldsymbol{F}_n}{m}$$

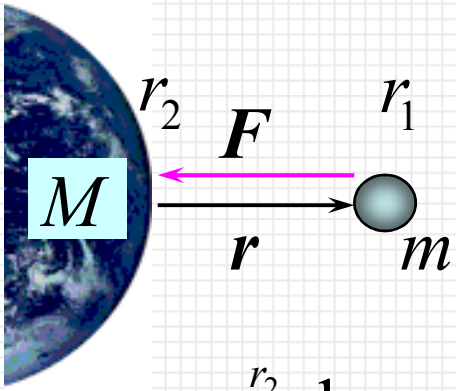
$$= \frac{\boldsymbol{F}_1}{m} + \frac{\boldsymbol{F}_2}{m} + \ldots + \frac{\boldsymbol{F}_n}{m} = \boldsymbol{K}_1 + \boldsymbol{K}_2 + \ldots + \boldsymbol{K}_n$$

$$\boldsymbol{K} = \sum_{i=1}^{n} \boldsymbol{K}_i \qquad = -\kappa \sum \frac{m_i}{r_i^3}\boldsymbol{r}_i \qquad = -\kappa \int \frac{\mathrm{d}m}{r^3}\boldsymbol{r} \qquad = -\kappa \int \frac{\boldsymbol{r}}{r^3}\rho \mathrm{d}\tau$$

Práca v gravitačnom poli, potenciálna energia

$E_p = ?$

$E_p = -A$



z Newtonovho gravitačného zákona

$$E_p = -\int_{r_1}^{r_2} \boldsymbol{F}.\mathrm{d}\boldsymbol{r} = \kappa m M \int_{r_1}^{r_2} \frac{\boldsymbol{r}.\mathrm{d}\boldsymbol{r}}{r^3} = \kappa m M \int_{r_1}^{r_2} \frac{\mathrm{d}\boldsymbol{r}}{r^2}$$

$$E_p = \kappa m M \left[-\frac{1}{r} \right]_{r_1}^{r_2}$$

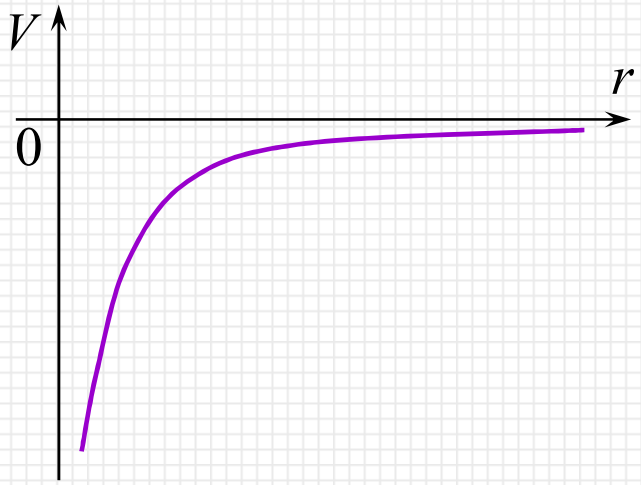
$$E_p = -\kappa m M \left(\frac{1}{r_2} - \frac{1}{r_1} \right)$$

$$A = E_p = -\kappa m M \left(\frac{1}{r_2} - \frac{1}{r_1} \right) \xrightarrow{r_1 \to \infty} -\frac{\kappa m M}{r} + 0$$

$$E_p = -\kappa m M \frac{1}{r}$$

Potenciál gravitačného poľa

$$V = \frac{E_p}{m} \quad \Rightarrow \quad V = -\kappa \frac{M}{r}$$



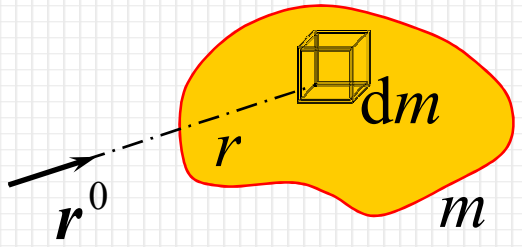
$$[V] = \frac{\mathrm{J}}{\mathrm{kg}} = \frac{\mathrm{kgm^2s^{-2}}}{\mathrm{kg}} = \mathrm{m^2s^{-2}}$$

pre sústavu hm. b. platí:

$$V = -\kappa \sum_i \frac{m_i}{r_i}$$

pre teleso so spojite rozloženou hmotnosťou platí:

$$V = -\kappa \int \frac{\mathrm{d}m}{r}$$



Gravitačné zrýchlenie

$$\boldsymbol{F} = m\boldsymbol{K} \Leftrightarrow \boldsymbol{F}_g = m\boldsymbol{g}$$

$$m\boldsymbol{K} = m\boldsymbol{g}$$

$$\boldsymbol{K} = \boldsymbol{g}$$

$$\boldsymbol{K} = -\kappa \frac{M}{r^3}\boldsymbol{r} \qquad = \boldsymbol{g}$$

$$g_h = \kappa \frac{M}{(R+h)^2}$$

vo výške h nad zemským povrchom

$$g_0 = \kappa \frac{M}{R^2}$$

na povrchu Zeme

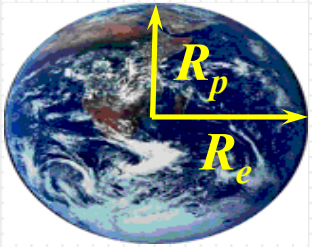
$$g_{Zem} = -6.671.10^{-11}\, N kg^{-2} m^2 \frac{5.974.10^{24}\, kg}{(6.371.10^6\, m)^2} = 9.818 ms^{-2}$$

$$\frac{g_{Zem}}{g_{Mesiac}} = 6.05$$

$$g_{Mesiac} = 1.623 ms^{-2}$$

Gravitačné zrýchlenie na rovníku a na póloch

$$M = 5{,}974.10^{24} kg$$
$$R_e = 6{,}38.10^6 m$$
$$R_p = 6{,}36.10^6 m$$



$$g_p = \kappa \frac{M}{R_p^2}$$

$$g_p = 6{,}671.10^{-11} Nm^2 kg^{-2} \frac{5{,}974.10^{24} kg}{(6{,}36.10^6 m)^2}$$

$$g_p = 9{,}86 ms^{-2}$$

$$F_{op} = 0$$

? odstredivá sila

$$g`_p = 9{,}86 ms^{-2}$$

$$g_e = \kappa \frac{M}{R_e^2}$$

$$g_e = 6{,}671.10^{-11} Nm^2 kg^{-2} \frac{5{,}974.10^{24} kg}{(6{,}38.10^6 m)^2}$$

$$g_e = 9{,}80 ms^{-2}$$

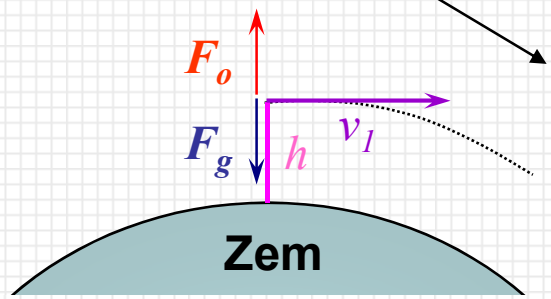
$$F_{oe} = \frac{4\pi^2 R_e}{T^2}$$

$$g'_e = g_e - \frac{4\pi^2 R_e}{T^2} = 9{,}80 ms^{-2} - \frac{39{,}48.6{,}38.10^6 m}{86400^2 s^2} = 9{,}77 ms^{-2}$$

Pohyby v gravitačnom poli Slnka – 1. kozmická rýchlosť

Kozmické rýchlosti

1. kozm. rýchlosť – obežnica Zeme
2. kozm. rýchlosť – opustenie gr. poľa Zeme
3. kozm. rýchlosť – opustenie Slnečnej sústavy



$$g = \kappa \frac{M}{(R+h)^2} \quad \Leftrightarrow \quad g_0 = -\kappa \frac{M}{R^2}$$

$$g = g_0 \frac{R^2}{(R+h)^2}$$

$$F_g = F_o$$

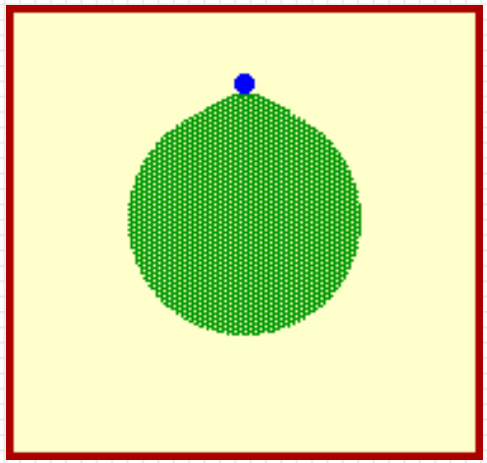
$$mg = m \frac{v_1^2}{R+h}$$

$$g_0 \frac{R^2}{(R+h)^2} = \frac{v_1^2}{R+h}$$

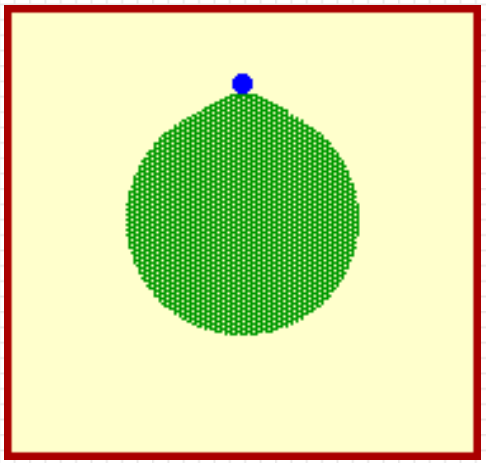
na povrchu Zeme $h = 0$

$$v_1 = R\sqrt{\frac{g_0}{R+h}} \quad \Rightarrow \quad v_1 = \sqrt{Rg_0} = 7.912 kms^{-1}$$

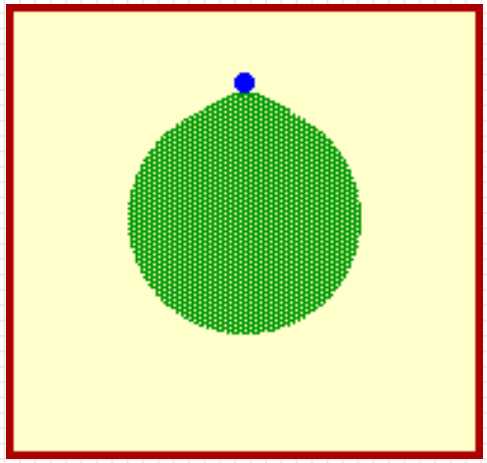
Obežné dráhy pre rôzne rýchlosti



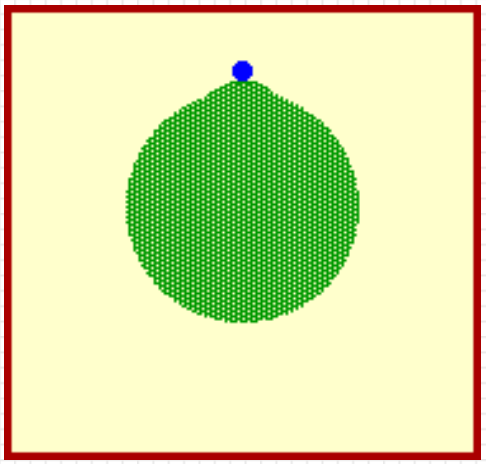
$v = 4kms^{-1}$



$v = 6kms^{-1}$



$v = 8kms^{-1}$



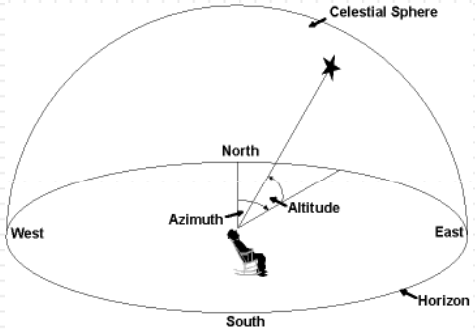
$v > 8kms^{-1}$

source: http://www.glenbrook.k12.il.us/gbssci/phys/mmedia/vectors/sat.html

Date	Mag	Starts			Max. altitude			Ends		
		Time	Alt.	Az.	Time	Alt.	Az.	Time	Alt.	Az.
17 Mar	-1.9	19:27:42	10	W	19:30:17	30	SSW	19:31:05	25	S
18 Mar	-3.1	18:16:57	10	WNW	18:19:49	66	SSW	18:22:40	10	ESE
18 Mar	-0.6	19:53:10	10	WSW	19:54:34	13	SW	19:54:59	13	SSW
19 Mar	-1.7	18:41:39	10	W	18:44:13	29	SSW	18:46:46	10	SSE
20 Mar	-0.3	19:07:05	10	WSW	19:08:24	12	SW	19:09:42	10	SSW
22 Mar	-0.1	18:20:51	10	WSW	18:22:02	12	SW	18:23:13	10	SSW

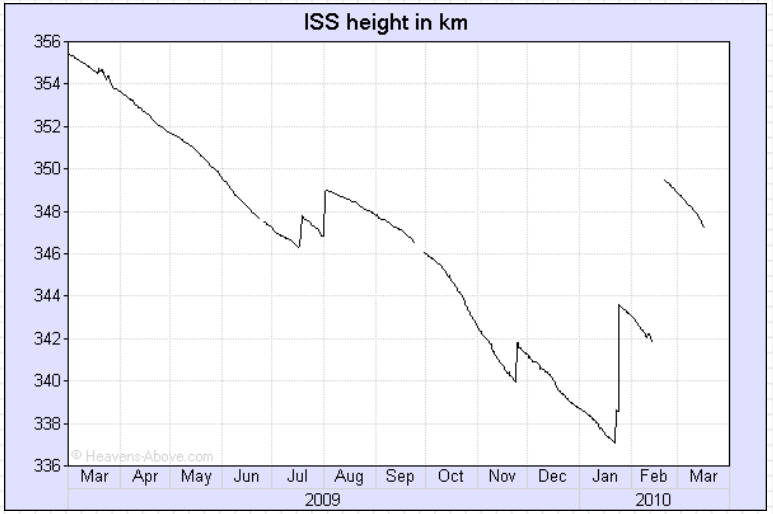
location: Zilina, 49.2170°N, 18.7330°E

N (0°), NNE (22.5°), NE (45°), ENE (67.5°), E (90°), ESE (112.5°), SE (135°), SSE (157.5°), S (180°), SSW (202.5°), SW (225°), WSW (247.5°), W (270°), WNW (292.5°), NW (315°), NNW (337.5°)

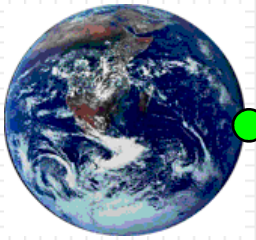


Magnitude

Sun (about 400 000 times brighter than full Moon!)	-26.7
Full Moon	-12.7
Brightest Iridium flares	-8
Venus (at brightest)	-4.4
International Space Station	-2
Sirius (brightest star)	-1.44
Limit of human eye	+6 to +7
Limit of 10x50 binoculars	+9
Pluto	+14
Limit of Hubble Space Telescope	+30



Pohyby v gravitačnom poli Slnka – 2. kozmická rýchlosť



$E_k + E_p$

$h = 0$ $\quad$ $h = \infty$

0 $\quad$ 0

$$E_{k\infty} + E_{p\infty} = E_{k0} + E_{p0}$$

$$E_{p\infty} = \kappa \frac{mM}{R}$$

$$E_{k0} = \frac{1}{2} m v_2^2$$

$$\kappa \frac{mM}{R} = \frac{1}{2} m v_2^2$$

$$v_2 = \sqrt{\frac{2\kappa M}{R}}$$

$$= 11.2 kms^{-1}$$

$v_{equator} = 0.5 kms^{-1}$ rýchlosť rotácie Zeme na rovníku

$v_{aroundSun} = 29.8 kms^{-1}$ rýchlosť rotácie Zeme okolo Slnka

$v_{Milkyway} = 250 kms^{-1}$ pohyb Slnečnej sústavy v galaxii Mliečna dráha